Sample B

Criteria	Teachers' mark
Personal Engagement	1
Exploration	3
Analysis	5
Evaluation	4
Communication	3
Total	16

An experiment to determine the value of Young's modulus of a meter ruler at room temperature by using a cantilever structure

DEFINITION: "Young's modulus, also known as the **tensile modulus** or **elastic modulus**, is a measure of the stiffness of an elastic material and is a quantity used to characterize materials. It is defined as the ratio of the stress (force per unit area) along an axis to the strain (ratio of deformation over initial length) along that axis in the range of stress in which Hooke's law holds.

From https://en.wikipedia.org/wiki/Young%27s_modulus

Introduction

A ruler is placed on the table such that one side is clamped on the table while the other is suspended in air, a cantilever. The part of the ruler that extends out from the table bends as the weights are added at the end of the ruler. However, what determines how much the ruler will bend is based on the weight applied? This relates to the Young's modulus. The Young's modulus is a ratio to determine the stiffness or tensile of a material. Different materials have different stiffness and those which are stiffer probably react less to the loads. Young's modulus is presented with the symbol "E".

The formula given to find Young's modulus: $M = \left(\frac{bW^3E}{4gL^3}\right)(y)$

M – Mass of the weights
b – Breadth of the meter ruler
W – Thickness of meter ruler
E – Young's modulus
g – Acceleration due to gravity
L – Length of the ruler which is overhanging from the table
y – Vertical height that the ruler bends

Independent Variable

The mass added on the suspended part of the ruler.

- Since the hook and slotted masses, are subjected to gravity, they apply a force downwards to the ruler as it is added to the edge of the meter ruler. Therefore, these are weights. The weight is applied on the meter ruler by tying the hook with a thread while the other end of the thread is tied at the edge of the meter ruler. To alter the weightage, more slotted mass are added to increase the weightage to get different readings for the experiment. The difference in weight should cause different amount of bending.

Dependent Variable

The bending of the ruler

- As the weights are added, the ruler will bend. The value of the bend is determined by indicating the difference in vertical height of the tip of the ruler to the ground before the load is added with the height of the tip of the ruler to the ground after the weight is added. When the bending occurs, the height of the tip of the ruler from the ground will instantly decrease. Record the new height after the weight is added. By subtracting the initial height to with the final height, the change in height will be found. This change in height is the amount of bending and the value that is to be found and compared to the weight that is added. Another ruler which is held up vertically by a retort stand is needed to be placed exactly at the edge of the horizontal meter ruler which is the cantilever. A toothpick is also placed at the edge of the cantilever to indicate on the vertical ruler the initial and final height.

Controlled Variable

1. The metre ruler- Different meter rulers have different value for Young's modulus, therefore, by changing the meter ruler, we would not be able to find the Young's modulus for that specific one meter ruler.
2. The length of the overhanging part of the ruler (**L**)- Based on the equation given, if we want to find the Young's modulus, **L** has to be kept constant as a change in **L** will cause a difference in readings. When finding the value for Young's modulus with the formula, **L** would also have to be found and applied to the formula for each reading.
3. The initial height of the meter ruler from the ground before the weight is added- If the initial height of the meter ruler from the ground is different from the initial readings or bended then it means that perhaps the meter ruler's Young's modulus have changed as it has deformed and is not able to be the same straight meter ruler as before.
4. The position of the vertical meter ruler which is held by the retort stand should remain unchanged. At different points of the ruler, the amount of bend or change in height is different. For example, the point on the meter ruler which is closer to the table would bend less than the point at the edge of the ruler. Therefore, if the position of the vertical meter ruler is changed and placed closer to the table, then the readings would no longer be consistent and accurate. Ensure that the toothpick which is placed on the edge of the meter ruler is always pointing towards the vertical meter ruler. Apart from that the angle of the vertical ruler from the ground has to always be the same which is 90° to ensure perpendicularity.
5. The position of the toothpick at the edge of the ruler. If the toothpick was not placed firmly and securely, the toothpick will move about. When this happens, the difference of the initial height and the final height would not be accurate.
6. Room temperature has to be a constant. Young's modulus differs depending on the temperature. Therefore, since the focus of the question is to find out the Young's modulus of the meter ruler at room temperature, the temperature has to be a constant. Since surrounding temperature fluctuates, the air conditioner is turned on to ensure that there is a consistency in temperature.

How to Control Variables

1. During all the experiments, the meter ruler is not removed or changed with another one. The same ruler which we are finding the Young's modulus is used throughout all the experiments.
2. Tightly secure the position of the ruler on the table with the 2 g-clamps to ensure that it does not move about. During all the experiments, the g-clamps are not removed and left there to avoid a change in the **L** (length of ruler which is overhanging from the table).
3. Ensure that after every experiment, the ruler returns to the same height as it was initially by just observing the height of the horizontal meter ruler on the vertical meter ruler before starting the experiments. Avoid leaving the weights on the edge of the meter ruler for too long as that may cause the deformation. After readings are taken, instantly remove the weights. If it has shown signs of deformity through the difference in initial height, then the whole experiment would have to be done again with a new ruler.
4. Before the start of the experiment, ensure that the toothpick is directly pointing on the vertical meter ruler. Always be careful not to push the retort stand or change the

position of the vertical meter ruler. The angle of the vertical meter ruler is also checked before the experiment is performed by using a set square and placing it on the meter ruler and ensuring there are no gaps between the vertical meter ruler and the set square at all 4 edges.

5. The toothpick has to be secured in position by using the cellophane tape to go over the toothpick and press the sides of the toothpick and the cellophane tape firmly so that there is no gaps between the cellophane tape and the ruler or toothpick. Its best to ensure that the toothpick is parallel to the ground.

Qualitative prediction

When weights are added, the lengths of the meter ruler nearer the table will not bend as much as the lengths of the ruler at the edge.

Quantitative prediction

The bend of the meter ruler will be proportional to the weights which are added at the edge of the ruler up to a certain point which is the elastic limit. Once the weight gets too heavy and exceeds the elasticity limit, the limit of proportionality will also be exceeded. At this point, the ruler no longer bends proportionally to the weight added. However, the values and readings from the experiment which is applied to the formula below will give the value for Young's modulus.

$$E = \frac{M4gL^3}{bW^3y}$$

E – Young's modulus
M – Mass of the weights
b – Breadth of the meter ruler
W – Thickness of meter ruler
g – Acceleration due to gravity
L – Length of the ruler which is overhanging from the table
y – Vertical height that the ruler bends

Apparatuses

- Meter ruler (one overhanging, one placed vertically upwards)
- 2 G- clamps
- Toothpick
- Retort stand clamp
- Hook
- Slotted masses
- Thread
- Electronic top pan balance
- Cellophane tape
- Thermometer

Method

1. Measure the temperature of the room with a thermometer. That will be the value of room temperature at which the Young's modulus is to be found.
2. Tie a thread at the hole located at the edge of the ruler. In this case, the hole nearest to the edge is the 1 cm point.
3. Place the meter ruler on the table horizontally flat such that the total overhanging length of the ruler is 51 cm. This is because the hole where the thread is tied to is at the 1 cm point. This would mean that "**L**" remains at 50 cm.
4. Clamp the ruler at that position with 2 g-clamps. This is to ensure that the ruler is securely in place.
5. Clamp another meter ruler on the retort stand clamp such that it is standing up vertically, perpendicular to the floor. Ensure that the 0cm point of the ruler touches the floor. To ensure that it is perpendicular, a set square is used by placing it against all 4 edges of the vertical meter ruler and ensure that there is no gap, hence defining that it is perpendicular to the ground.
6. Secure a toothpick at the edge of the overhanging part of the metre ruler with cellophane tape.
7. Place the vertical meter ruler with the retort stand near the edge of the horizontal meter ruler such that the toothpick points and touches towards the measuring scales of the vertical ruler. As the toothpick touches the measuring scale, it will reduce parallax error.
8. Record the initial height of the horizontal meter ruler from the ground.
9. Weigh the hook on the top pan balance and record the reading.
10. Hang the hook on the thread
11. Measure the current height of the horizontal ruler from the ground based on the toothpick.
12. Record the new height and remove the weights in order to avoid the ruler from permanent deformity.
13. Subtract the initial height with the final height after the hook is added. Record this value.
14. Repeat step 7-11 three times
15. Next add some slotted masses of 100g to the hook and weigh it on the top pan balance. Record the actual mass.
16. Place the hook with added slotted masses on the thread and identify the new height of the horizontal meter ruler to the ground. Again, subtract the initial height with the final height after the weight is added and record this reading.
17. Then go on with the experiment and change the masses by 100g for every experiment. Ensure that for all masses, 3 repeats are done.

Raw Data

Uncertainty:

- Minimum scale of thermometer= 0.1°C
  Uncertainty of measurement of temperature = $\pm\left(\frac{1}{2}\times 0.1\ ^{\circ}\mathrm{C}\right)$
  = $\pm 0.05^{\circ}\mathrm{C}$
- Minimum scale of the metre ruler = 0.1cm
  Uncertainty of measurements with the metre ruler = $\pm\left(\frac{1}{2}\times 0.1\ cm\right)$
  = ± 0.05 cm
- Minimum scale of the digital top pan balance = 0.01 g
  Since it is digital device, the uncertainty = ± 0.01g
- Minimum scale of vernier calliper = 0.01 cm
  Uncertainty of measurements with vernier calliper = $\pm\left(\frac{1}{2}\times 0.01 cm\right)$
  = ± 0.005 cm
- Calculation of measurement using vernier calliper :
  V.C =1MSD – 1VSD

$$1\text{MSD} - 9/10\ \text{MSD} = \frac{1}{10}\ \text{x MSD}$$
$$= \frac{1}{10}\ \text{x 1mm}$$
$$= 0.1\text{mm}$$
$$= 0.01\text{cm}$$

Example:
Breadth of meter ruler = MSR + V.C. x VSR
= 2.8 + (0.01 x 9)

= 2.830 cm ± 0.005cm

Table of raw data collected

Observation	Room temperature (°C) $\pm 0.05^{\circ}\mathrm{C}$	Mass (g) ±0.01 g	Height of the edge of ruler from ground after load is added (cm) ±0.05cm		
			1st reading	2nd reading	3rd reading
1	25.30	100.00	88.10	88.10	88.00
2		201.82	87.10	87.10	87.10
3		302.56	86.20	86.20	86.10
4		403.58	85.30	85.30	85.20
5		504.90	84.30	84.20	84.30

Length of overhanging part of ruler (L) = 50.00 cm± 0.05 cm
Thickness of ruler (W) = 0.640 cm± 0.005 cm
Breadth of ruler (b) = 2.890cm ± 0.005 cm
Height of the table initially = 89.00 cm ± 0.05 cm
Acceleration due to gravity= 9.81 m/s^2 [1] or 9.81N/kg
* The value of acceleration due to gravity varies by location but this is the average value found.

Uncertainty is $\pm 0.005cm$ because vernier calliper is used

Uncertainty is $\pm 0.05cm$ because metere ruler

[1] http://www.aerospaceweb.org/question/dynamics/q0203.shtml

Data Processing

Processed data table

Observation	Mass (g) ±0.01 g	Bend of the ruler (cm) $\pm 0.1cm$			Average bend of ruler (cm)	Uncertainty of average bend of ruler (cm)
		1st reading	2nd reading	3rd reading		
1	100.00	0.9	0.9	1.0	0.93	± 0.05
2	201.82	1.9	1.9	1.9	1.90	±0.00
3	302.56	2.8	2.8	2.9	2.83	± 0.05
4	403.58	3.7	3.7	3.8	3.73	± 0.05
5	504.90	4.7	4.8	4.7	4.73	± 0.05

Example taken from observation 1:

- Uncertainty of the bend of the ruler, the change in height after the load has be added
  = uncertainty of initial height + uncertainty of the height after the load are added
  = $\pm$(0.05 cm + 0.05 cm)
  = ±0.1 cm
  *since the reading of the bend comprises of the initial height and the final height after the load is added, there will be an addition of uncertainty twice because the initial height is found through the scale on the ruler. At this point, there will be an uncertainty of $\pm 0.05cm$. Then, the height after load is added is found through the scale again which is another $\pm 0.05cm$.
- The bend of the ruler, the change in height after the load has been added, y
  = initial height – final height
  = 89.00 cm – 88.10 cm
  =0.90 cm ± 0.10 cm

- Uncertainty of the average bend of the ruler
  $= \frac{max-min}{2}$
  $= \frac{1.0-0.9}{2}$
  $= \pm\ 0.05\ cm$

- Average bend of the ruler
  $= \frac{1st+2nd+3rd\ reading}{3}$
  $= \frac{0.9+0.9+1.0}{3}$
  = 0.93cm $\pm\ 0.05\ cm$

In order to find the Young's modulus of the meter ruler, the formula can be rewritten:

$$E = \frac{M4gL^3}{bW^3y}$$

$$\text{Let } \mathrm{bW^3y} = \alpha \text{ and } \mathrm{M4gL^3} = \beta$$

$$\alpha = \frac{\beta}{E}$$

Therefore, the data will be presented in terms of α *and* β and the values of both α *and* β will be plotted on a graph to find the value of Young's modulus. All lengths are converted from centimetres to metres and all mass are converted from grams to kilograms. β will be plotted on the x-axis because mass is the independent variable in this experiment.

Table showing the values of α and β

Observation	α (m^5) $\times 10^{-2}$	Percentage uncertainty of α	Uncertainty of α (m^5) $\times 10^{-3}$	β (Nm^3) $\times 10^{6}$	Percentage uncertainty of β	Uncertainty of β (Nm^3) $\times 10^{3}$
1	0.71	$\pm 9.45\%$	± 0.67	0.49	$\pm 0.31\%$	± 1.52
2	1.44	$\pm 4.07\%$	± 0.59	0.99	$\pm 0.30\%$	± 3.02
3	2.14	$\pm 5.84\%$	± 1.25	1.48	$\pm 0.30\%$	± 4.50
4	2.83	$\pm 5.41\%$	± 1.53	1.98	$\pm 0.30\%$	± 5.99
5	3.58	$\pm 5.13\%$	± 1.84	2.48	$\pm 0.30\%$	± 7.48

Example taken from observation 1:

- Percentage Uncertainty of $\propto$

  $= \left[\left(\frac{uncertainty\ of\ b}{b}\right) + (3)\left(\frac{uncertainty\ of\ w}{w}\right) + \left(\frac{uncertainty\ of\ y}{y}\right)\right] \times 100\%$

  $= \left[\left(\frac{\pm 0.005}{2.89}\right) + (3)\left(\frac{\pm 0.005}{0.64}\right) + \left(\frac{\pm 0.05}{0.93}\right)\right] \times 100\%$

  $= \pm 9.45\%\ (2d.p.)$

- Uncertainty of $\propto$

  $= (percentage\ uncertainty\ of\ \propto) \times (\propto)$

  $= \pm 9.45\% \times 0.00704564$

  $= \pm 0.000666\ \mathrm{m^5}$

  $= \pm 0.67 \times 10^{-3}\ m^5 (2d.p.)$

- Calculation of $\alpha = bW^3y$

$$= 0.0289 \times (0.0064)^3 \times 0.0093$$
$$= 0.00704564\text{m}^5$$
$$= (0.71 \times 10^{-2} m^5) \pm (0.67 \times 10^{-3}\ m^5)\ (2d.p.)$$

- Percentage Uncertainty of β

$$= \left[\left(\frac{uncertainty\ of\ M}{M}\right) + (3)\left(\frac{uncertainty\ of\ L}{L}\right)\right] \times 100\%$$
$$= \left[\left(\frac{\pm 0.01}{100}\right) + (3)\left(\frac{\pm 0.05}{50}\right)\right] \times 100\%$$
$$= \pm 0.31\%\ (2d.p.)$$

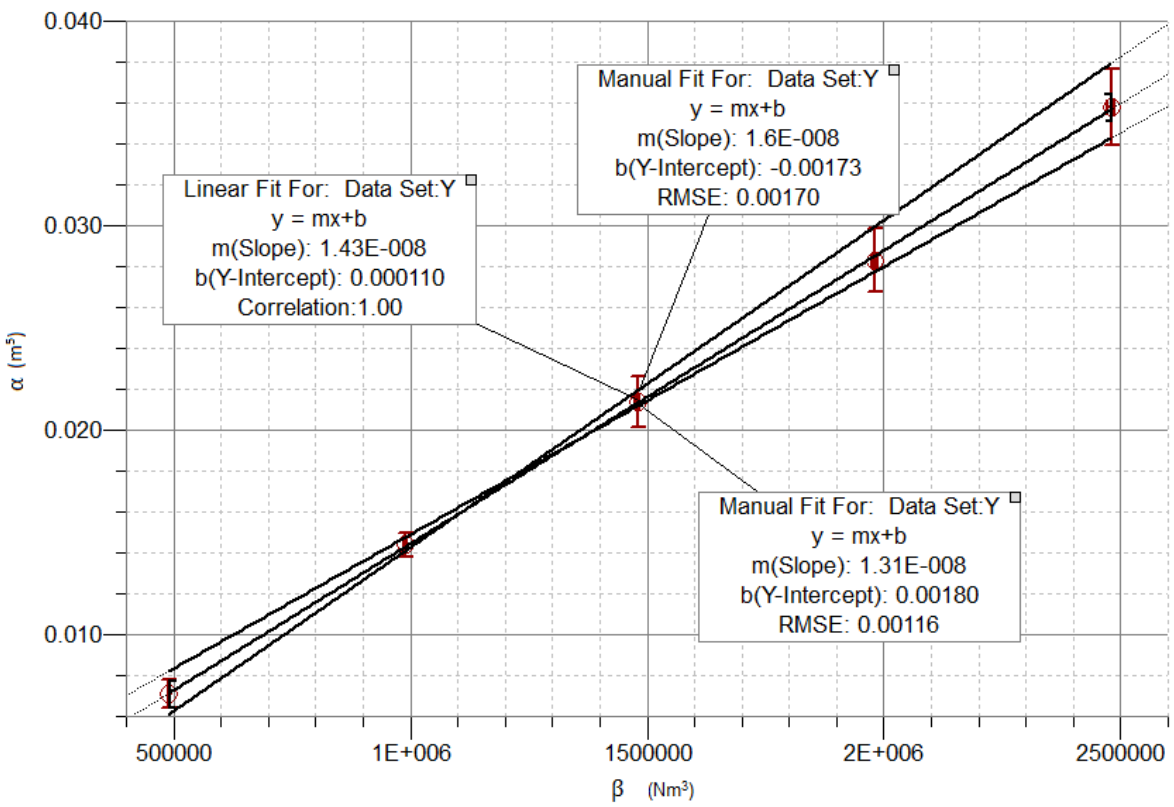
- Uncertainty of β

$$= (percentage\ uncertainty\ of\ \beta) \times (\beta)$$
$$= \pm 0.31\% \times 490500$$
$$= \pm 1520.55\ \text{Nm}^3$$
$$= \pm 1.52 \times 10^3\ Nm^3 (2d.p.)$$

- Calculation of $\beta = M4gL^3$

$$= 0.1 \times 4 \times 9.81 \times (50)^3$$
$$= 490500\ \text{Nm}^3$$
$$= (0.49 \times 10^6\ Nm^3) \pm (1.52 \times 10^3\ Nm^3)(2d.p.)$$

Graph of α against β



Conclusion

The graph of β against α appears to be a straight line. This shows the direct proportional relationship between the two variables. As β increases, so will α. The variable of β is the mass added to the ruler because every other value used to form β is a constant. The variable for α is the bend of the ruler as again every other value used to form α is a constant.

The uncertainty of the gradient of the graph is:

$\pm\left(\frac{maximum\ gradient - minimum\ gradient}{2}\right)$

$=\pm\left(\frac{(1.6\times10^{8})-(1.31\times10^{8})}{2}\right)$

$=\pm1.45\times10^{7}Nm^{-2}$

Therefore, based on the graph, the Young's modulus at room temperature 25.3℃ is:

$$E = \frac{1}{gradient\ of\ graph}$$

$$= \frac{1}{1.43\times10^{-8}}$$

$$= 6.99\times10^{7}Nm^{-2} \pm 1.45\times10^{7}Nm^{-2}$$

However, the Young's modulus value is only for this ruler. A different ruler may have varying value for Young's modulus.

Random Errors

1) It was difficult to take the readings of the change in height as the toothpick and the weights often move about and it is not fixed at the centre. Besides, there could easily be parallax error as the toothpick is not situated on the meter ruler itself, it is somewhat a distance away, and therefore, the slightest angle of sight away from the perpendicular will cause a huge change difference from the actual height.
2) As the ruler has been subjected to the heavy weights for long periods, this may cause the ruler to change its shape or deform which means that the Young's modulus will change through the course of taking readings.
3) The measuring ruler which was placed vertically upwards could be at an angle. It may not be accurately 90° or perpendicular to the ground. Even if there were no parallax error and reading were cautiously observed, the reading may not be very accurate because of the slant of the ruler.
4) The value of "g" is different depending on the location. In this experiment to calculate the Young's modulus, we took a general figure of "g" which is 9.81N/kg. However, this may not be the case at where the experiment was performed.
5) The temperature of the classroom may not be a constant even with the air-conditioner on because air-conditioners work with the usage of a thermostat and when the temperature starts to rise, then only it starts running faster so that the room temperature falls back. This means that the temperature will not be a constant all throughout. The reading of temperature was only taken once. Young's modulus changes with temperature. Therefore, taking readings at different intervals may result to different temperatures which also mean that different Young's modulus is found at different time.

Systematic Errors

1) There is a possibility of the meter ruler having inaccurate calibration of the scales. However, even if this is so, we would not be able to tell at the instant look that the scales are not right. In order to avoid this from happening, it is best to test and check the ruler to ensure that it has the correct readings.
2) The weighing balance may also have errors in the calibration. This means that the mass that is quoted is not necessarily the actual mass of the slotted masses. This will affect the value of Young's modulus found as that is one of the variables.

Methods of improving experiment

Random Errors

1) In order to minimize the error in identifying the change in height of the ruler, we have to bend down till the eye level is exactly on par with the toothpick. Read the reading on the vertical measuring ruler by observing the scale which is covered by the toothpick. It is best to be able to locate the measuring ruler in a way that the toothpick lays directly on the vertical measuring ruler itself. This will further eliminate parallax error.
2) To avoid deformity of the meter ruler, take the readings quickly and remove the weights before the next weights are added. This will reduce the tension that the ruler is subjected to for long periods of time, therefore, reducing the deformity caused by the weights.
3) To ensure that the vertical measuring ruler is perpendicularly straight, place a block of a wood or a set square on each side of the ruler till there is no gaps between the measuring ruler and the tools. Once that is so, then it is perpendicular. This could be done by pushing the ruler till there is no more gap or pulling it back. Readjust till it is directly perpendicular to the ground.
4) In order to have a very accurate experiment, the value of "g" should be calculated at the position where experiment is conducted. This could be done with a bar pendulum. However, finding the value of "g" in the school lab will contain lots of uncertainties and inaccuracies as well.
5) Instead of just switching on the air-conditioner, give some time for the room temperature to stabilize. Ensure that indeed the temperature does not change. Immediately switching on the air-conditioner will not affect the room temperature instantaneously. Besides, our body temperature itself may affect a change in temperature but all these differences in temperature are very minimal and often negligible. Perhaps a good way to reduce the change in temperature of the ruler is by painting the ruler with a silver coating that will reflect heat. However, again painting the ruler may affect the Young's modulus as the physical properties on the outside may change.

Systematic Errors

1) Cross checking the scales of the meter ruler with a different ruler of perhaps a different brand. If the scales are the same, then there is no problem. However, it is not likely that the difference is large. It is often negligible.
2) Again it is best to check the mass with a different weighing balance to confirm that the weighing balance indeed gives accurate readings.